



Lesson 18: Distance on the Coordinate Plane

Student Outcomes

- Students compute the length of horizontal and vertical line segments with integer coordinates for end points in the coordinate plane by counting the number of units between end points and using absolute value.

Classwork

Opening Exercise (5 minutes)

Opening Exercise

Four friends are touring on motorcycles. They come to an intersection of two roads; the road they are on continues straight, and the other is perpendicular to it. The sign at the intersection shows the distances to several towns. Draw a map/diagram of the roads, and use it and the information on the sign to answer the following questions:

Albertsville ← 8 mi.

Blossville ↑ 3 mi.

Cheyenne ↑ 12 mi.

Dewey Falls → 6 mi.

What is the distance between Albertsville and Dewey Falls?

Students draw and use their maps to answer. Albertsville is 8 miles to the left, and Dewey Falls is 6 miles to the right. Since the towns are in opposite directions from the intersection, their distances must be combined by addition, $8 + 6 = 14$, so the distance between Albertsville and Dewey Falls is 14 miles.

What is the distance between Blossville and Cheyenne?

Blossville and Cheyenne are both straight ahead from the intersection in the direction that they are going. Since they are on the same side of the intersection, Blossville is on the way to Cheyenne, so the distance to Cheyenne includes the 3 miles to Blossville. To find the distance from Blossville to Cheyenne, I have to subtract; $12 - 3 = 9$. So, the distance from Blossville to Cheyenne is 9 miles.

On the coordinate plane, what represents the intersection of the two roads?

The intersection is represented by the origin.

Example 1 (6 minutes): The Distance Between Points on an Axis

Students find the distance between points on the x -axis by finding the distance between numbers on the number line. They find the absolute values of the x -coordinates and add or subtract their absolute values to determine the distance between the points.

Example 1: The Distance Between Points on an Axis

Consider the points $(-4, 0)$ and $(5, 0)$.

What do the ordered pairs have in common, and what does that mean about their location in the coordinate plane?

Both of their y -coordinates are zero, so each point lies on the x -axis, the horizontal number line.

How did we find the distance between two numbers on the number line?

We calculated the absolute values of the numbers, which told us how far the numbers were from zero. If the numbers were located on opposite sides of zero, then we added their absolute values together. If the numbers were located on the same side of zero, then we subtracted their absolute values.

Use the same method to find the distance between $(-4, 0)$ and $(5, 0)$.

$|-4| = 4$ and $|5| = 5$. The numbers are on opposite sides of zero, so the absolute values get combined: $4 + 5 = 9$. The distance between $(-4, 0)$ and $(5, 0)$ is 9 units.

Example 2 (5 minutes): The Length of a Line Segment on an Axis

Students find the length of a line segment that lies on the y -axis by finding the distance between its end points.

Example 2: The Length of a Line Segment on an Axis

Consider the line segment with end points $(0, -6)$ and $(0, -11)$.

What do the ordered pairs of the end points have in common, and what does that mean about the line segment's location in the coordinate plane?

The x -coordinates of both end points are zero, so the points lie on the y -axis, the vertical number line. If its end points lie on a vertical number line, then the line segment itself must also lie on the vertical line.

Find the length of the line segment described by finding the distance between its end points $(0, -6)$ and $(0, -11)$.

$|-6| = 6$ and $|-11| = 11$. The numbers are on the same side of zero, which means the longer distance contains the shorter distance, so the absolute values need to be subtracted: $11 - 6 = 5$. The distance between $(0, -6)$ and $(0, -11)$ is 5 units, so the length of the line segment with end points $(0, -6)$ and $(0, -11)$ is 5 units.

Example 3 (10 minutes): Length of a Horizontal or Vertical Line Segment That Does Not Lie on an Axis

Students find the length of a vertical line segment that does not lie on the y -axis by finding the distance between its end points.

Example 3: Length of a Horizontal or Vertical Line Segment That Does Not Lie on an Axis

Consider the line segment with end points $(-3, 3)$ and $(-3, -5)$.

What do the end points, which are represented by the ordered pairs, have in common? What does that tell us about the location of the line segment on the coordinate plane?

Both end points have x -coordinates of -3 , so the points lie on the vertical line that intersects the x -axis at -3 . This means that the end points of the line segment, and thus the line segment, lie on a vertical line.

Find the length of the line segment by finding the distance between its end points.

The end points are on the same vertical line, so we only need to find the distance between 3 and -5 on the number line. $|3| = 3$ and $|-5| = 5$, and the numbers are on opposite sides of zero, so the values must be added: $3 + 5 = 8$. So, the distance between $(-3, 3)$ and $(-3, -5)$ is 8 units.

Scaffolding:

Students may need to draw an auxiliary line through the end points to help visualize a horizontal or vertical number line.

MP.7

Exercise (10 minutes)

Students calculate the distance between pairs of points using absolute values.

Exercise

Find the lengths of the line segments whose end points are given below. Explain how you determined that the line segments are horizontal or vertical.

a. $(-3, 4)$ and $(-3, 9)$

Both end points have x -coordinates of -3 , so the points lie on a vertical line that passes through -3 on the x -axis. $|4| = 4$ and $|9| = 9$, and the numbers are on the same side of zero. By subtraction, $9 - 4 = 5$, so the length of the line segment with end points $(-3, 4)$ and $(-3, 9)$ is 5 units.

b. $(2, -2)$ and $(-8, -2)$

Both end points have y -coordinates of -2 , so the points lie on a horizontal line that passes through -2 on the y -axis. $|2| = 2$ and $|-8| = 8$, and the numbers are on opposite sides of zero, so the absolute values must be added. By addition, $8 + 2 = 10$, so the length of the line segment with end points $(2, -2)$ and $(-8, -2)$ is 10 units.

c. $(-6, -6)$ and $(-6, 1)$

Both end points have x -coordinates of -6 , so the points lie on a vertical line. $|-6| = 6$ and $|1| = 1$, and the numbers are on opposite sides of zero, so the absolute values must be added. By addition, $6 + 1 = 7$, so the length of the line segment with end points $(-6, -6)$ and $(-6, 1)$ is 7 units.

d. $(-9, 4)$ and $(-4, 4)$

Both end points have y -coordinates of 4, so the points lie on a horizontal line. $|-9| = 9$ and $|-4| = 4$, and the numbers are on the same side of zero. By subtraction, $9 - 4 = 5$, so the length of the line segment with end points $(-9, 4)$ and $(-4, 4)$ is 5 units.

e. $(0, -11)$ and $(0, 8)$

Both end points have x -coordinates of 0, so the points lie on the y -axis. $|-11| = 11$ and $|8| = 8$, and the numbers are on opposite sides of zero, so their absolute values must be added. By addition, $11 + 8 = 19$, so the length of the line segment with end points $(0, -11)$ and $(0, 8)$ is 19 units.

Closing (3 minutes)

- Why is it possible for us to find the length of a horizontal or vertical line segment even if it's not on the x - or y -axis?
 - *A line can still be a horizontal or vertical line even if it is not on the x - or y -axis; therefore, we can still use the same strategy.*
- Can you think of a real-world situation where this might be useful?
 - *Finding the distance on a map*

Lesson Summary

To find the distance between points that lie on the same horizontal line or on the same vertical line, we can use the same strategy that we used to find the distance between points on the number line.

Exit Ticket (6 minutes)



Name _____

Date _____

Lesson 18: Distance on the Coordinate Plane

Exit Ticket

Determine whether each given pair of end points lies on the same horizontal or vertical line. If so, find the length of the line segment that joins the pair of points. If not, explain how you know the points are not on the same horizontal or vertical line.

a. $(0, -2)$ and $(0, 9)$

b. $(11, 4)$ and $(2, 11)$

c. $(3, -8)$ and $(3, -1)$

d. $(-4, -4)$ and $(5, -4)$

Exit Ticket Sample Solutions

Determine whether each given pair of end points lies on the same horizontal or vertical line. If so, find the length of the line segment that joins the pair of points. If not, explain how you know the points are not on the same horizontal or vertical line.

- a. $(0, -2)$ and $(0, 9)$

The end points both have x -coordinates of 0, so they both lie on the y -axis, which is a vertical line. They lie on opposite sides of zero, so their absolute values have to be combined to get the total distance. $|-2| = 2$ and $|9| = 9$, so by addition, $2 + 9 = 11$. The length of the line segment with end points $(0, -2)$ and $(0, 9)$ is 11 units.

- b. $(11, 4)$ and $(2, 11)$

The points do not lie on the same horizontal or vertical line because they do not share a common x - or y -coordinate.

- c. $(3, -8)$ and $(3, -1)$

The end points both have x -coordinates of 3, so the points lie on a vertical line that passes through 3 on the x -axis. The y -coordinates lie on the same side of zero. The distance between the points is determined by subtracting their absolute values, $|-8| = 8$ and $|-1| = 1$. So, by subtraction, $8 - 1 = 7$. The length of the line segment with end points $(3, -8)$ and $(3, -1)$ is 7 units.

- d. $(-4, -4)$ and $(5, -4)$

The end points have the same y -coordinate of -4 , so they lie on a horizontal line that passes through -4 on the y -axis. The numbers lie on opposite sides of zero on the number line, so their absolute values must be added to obtain the total distance, $|-4| = 4$ and $|5| = 5$. So, by addition, $4 + 5 = 9$. The length of the line segment with end points $(-4, -4)$ and $(5, -4)$ is 9 units.

Problem Set Sample Solutions

1. Find the length of the line segment with end points $(7, 2)$ and $(-4, 2)$, and explain how you arrived at your solution.

11 units. Both points have the same y -coordinate, so I knew they were on the same horizontal line. I found the distance between the x -coordinates by counting the number of units on a horizontal number line from -4 to zero and then from zero to 7, and $4 + 7 = 11$.

or

I found the distance between the x -coordinates by finding the absolute value of each coordinate. $|7| = 7$ and $|-4| = 4$. The coordinates lie on opposite sides of zero, so I found the length by adding the absolute values together. Therefore, the length of a line segment with end points $(7, 2)$ and $(-4, 2)$ is 11 units.

2. Sarah and Jamal were learning partners in math class and were working independently. They each started at the point $(-2, 5)$ and moved 3 units vertically in the plane. Each student arrived at a different end point. How is this possible? Explain and list the two different end points.

It is possible because Sarah could have counted up and Jamal could have counted down or vice versa. Moving 3 units in either direction vertically would generate the following possible end points: $(-2, 8)$ or $(-2, 2)$.

3. The length of a line segment is 13 units. One end point of the line segment is $(-3, 7)$. Find four points that could be the other end points of the line segment.

$(-3, 20)$, $(-3, -6)$, $(-16, 7)$ or $(10, 7)$